Efficient Joint Hybrid Precoding And Analog Combining Scheme For Massive MIMO Systems

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Contents

1 Background

Hybrid Precoding For Massive MIMO

- Role: An alternative to the traditional digital precoding
- Target: Improve the spectral efficiency

high hardware cost on radio frequency chains with the growing number of antennas

Challenge upon Hybrid Precoding

- The constant modulus constraint in the analog domain is non-convex constraint.
- The total transmit power constraint also needs to be met.
- The hybrid precoding algorithm needs to eliminate the inter-user interference for one certain user.
- The hybrid precoding algorithm needs to improve the spectral efficiency while guaranteeing computational complexity not too high.

1 Background

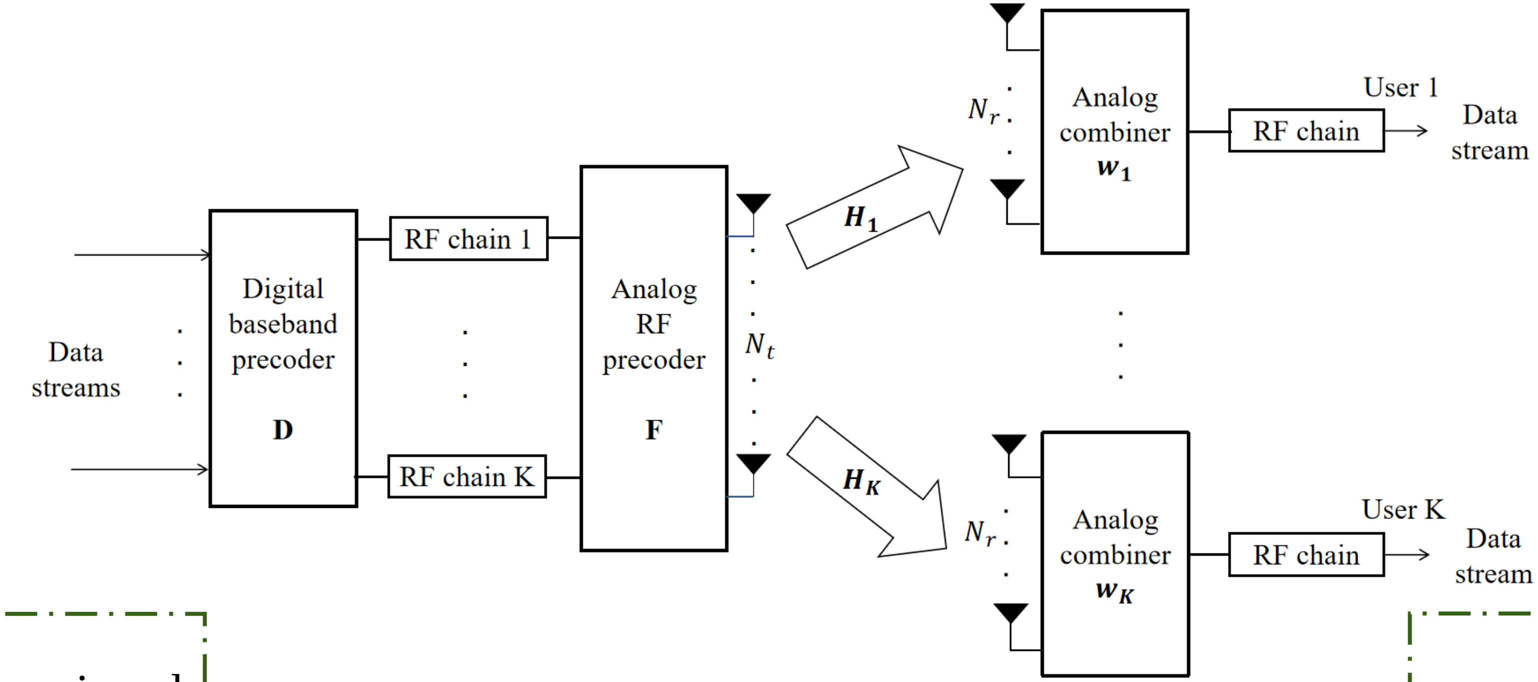
The existing hybrid precoding algorithms only consider the massive MIMO systems with single-antenna UE.

It influences the application in practice.	It ignores the spatial diversity and array gain in MIMO channel.
Most algorithms only focus on hybrid beamforming in MISO systems, which limits their applications.	This paper will prove that the analog combining scheme improves the spectral efficiency by extracting the array gain of multiple receive antennas.

Jointly design hybrid precoding and analog combining scheme.

2 System model

A multi-user massive MIMO downlink system.



The base station is equipped with N_t transmit antennas and K RF chains to transmit K data streams to K users.

$$\begin{aligned}\mathbf{y}_k &= \mathbf{w}_k^H(\mathbf{H}_k\mathbf{F}\mathbf{D}\mathbf{s}+\mathbf{n}_k)\\ &= \mathbf{w}_k^H\mathbf{H}_k\mathbf{F}\mathbf{D}s_k + \sum_{j\neq k}\mathbf{w}_k^H\mathbf{H}_k\mathbf{F}\mathbf{D}s_j + \mathbf{w}_k^H\mathbf{n}_k.\end{aligned}$$

Each user is equipped with $N_r > 1$ receive antennas and one RF chain

2 System model

The received signal-to-interference-plus-noise ratio (SINR) of the k-th user is given by:

$$\mathrm{SINR}_k = \frac{\frac{P}{K}|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{d}^k|^2}{1 + \sum_{j \neq k} \frac{P}{K}|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{d}^j|^2}$$

Given Gaussian inputs, the sum spectral efficiency turns out to be:

$$R = \sum_{k=1}^{K} \mathbb{E}\left[\log_2\left(1 + \mathrm{SINR}_k\right)\right].$$

The goal of hybrid precoding and analog combining is:

$$\max_{\mathbf{w}_k, \mathbf{F}, \mathbf{D}} \sum_{k=1}^{K} \mathbb{E}\left[\log_2\left(1 + \frac{\frac{P}{K}|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{d}^k|^2}{1 + \sum_{j \neq k} \frac{P}{K}|\mathbf{w}_k^H \mathbf{H}_k \mathbf{F} \mathbf{d}^j|^2}\right)\right]$$

In particular, the analog combiner can be obtained by solving:

$$\max_{\mathbf{w}_k} \sum_{k=1}^{K} \left\|\mathbf{w}_k^H \mathbf{H}_k\right\|_1^2,$$

$$s.t. \left|\mathbf{w}_k^H(i)\right| = \frac{1}{\sqrt{N_r}}, \forall i, k.$$

3 Hybrid precoding and analog combining design

Why considering analog combining scheme designed for multiple-antenna UE?

Theory Analysis

When $N_t \times N_r$ tends to infinity,
the theoretical spectral efficiency achieved by the joint hybrid precoding and analog combining in Rayleigh fading massive MIMO systems with multiple-antenna UE is:

$$\lim_{N_t \times N_r \to \infty} R = K \log_2 \left(1 + \frac{\pi}{4} \frac{P N_t N_r}{K}\right)$$

Proof

When the inter-user interference is eliminated, the sum spectral efficiency can be transformed to:

$$R = \sum_{k=1}^{K} \mathbb{E}\left[\log_2\left(1 + \frac{P}{K}|\mathbf{w}_k^H \mathbf{H}_k \mathbf{f}^k|^2\right)\right]$$

Given the optimum of phase eliminations $\mathbf{w}_k^*$ and $\mathbf{f}^{k*}$ we can obtain the following result:

$$\mathbf{w}_k^{*H} \mathbf{H}_k \mathbf{f}^{k*} = \frac{1}{\sqrt{N_t N_r}} \sum_{i=1}^{N_r} \sum_{l=1}^{N_t} \left|\mathbf{h}_k^l(i)\right|, \forall i, l,$$

As for Rayleigh fading channels, according to the central limit theorem, we have

$$\mathbf{w}_k^{*H} \mathbf{H}_k \mathbf{f}^{k*} \sim \mathcal{N}\left(\frac{\sqrt{\pi N_t N_r}}{2}, 1 - \frac{\pi}{4}\right)$$

Then the spectral efficiency can be derived as:

$$R = K\mathbb{E}\left[\log_2\left(1 + \frac{P}{K}\left(x + \frac{\sqrt{\pi N_t N_r}}{2}\right)^2\right)\right]$$

$$= K\log_2\left(1 + \frac{\pi}{4}\frac{P N_t N_r}{K}\right) + K\mathbb{E}\left[\log_2\left(\frac{1 + \frac{P}{K}\left(x + \frac{\sqrt{\pi N_t N_r}}{2}\right)^2}{1 + \frac{\pi N_t N_r}{4}\frac{P}{K}}\right)\right]$$

Intuitively, when $N_t \times N_r$ tends to infinity, the second term is limited to zero, completing the proof.

It only serves as an upper bound of the spectral efficiency, and how to approach it chiefly lies on the design of the analog combining and the analog precoding.

3 Hybrid precoding and analog combining design

How to approach the theoretical spectral efficiency?

Analog Combining Scheme

The optimization target of analog combining

We set $\mathbf{q}_k = \mathbf{w}_k^H \mathbf{H}_k \in \mathbb{C}^{1\times N_t}$

The problem can be transformed to maximize the modulus of each element in $\mathbf{q}_k$:

$$\mathbf{q}_k(l) = \frac{1}{\sqrt{N_r}} \sum_{i=1}^{N_r} |\mathbf{h}_k^l(i)| e^{j\theta_{i,l}} e^{j\phi_i}, \forall l$$

$\theta_{i,l}$ and ϕ_i represent the complex angles of $\mathbf{h}_k^l(i)$ and $\mathbf{w}_k^H(i)$.

Clearly, the modulus of $\mathbf{q}_k(l)$ can be maximized if and only if $\theta_{i,l} + \phi_i$ is small enough, which is known as phase elimination.

Let us denote the optimization target of $\mathbf{q}_k$ as $\mathbf{q}_k^*$

$$\mathbf{q}_k^*(l) = \frac{1}{\sqrt{N_r}} \sum_{i=1}^{N_r} |\mathbf{h}_k^l(i)|, \forall l.$$

In this way, the non-convex problem can be transformed to a least square problem with modulus constraint as followed:

$$\min_{\mathbf{w}_k} \sum_{k=1}^{K} \left\| \mathbf{q}_k^* - \mathbf{w}_k^H \mathbf{H}_k \right\|_2^2,$$

$$s.t. \left| \mathbf{w}_k^H(i) \right| = \frac{1}{\sqrt{N_r}}, \forall i, k.$$

The application of gradient descent projection

Given the r-th iteration $\mathbf{w}_k^{H(r)}$, the r+1-th iteration computes:

$$\zeta^{(r+1)} = \mathbf{w}_k^{H(r)} + \alpha(\mathbf{q}_k^* - \mathbf{w}_k^{H(r)}\mathbf{H}_k)\mathbf{H}_k^H;$$

$$\mathbf{w}_k^{H(r+1)} = \frac{1}{\sqrt{N_r}} e^{j\angle\zeta^{(r+1)}};$$

$$r = r + 1,$$

To aviod introducing extra computational complexity, here we set the initial setup as $\mathbf{w}_k^{H(0)} = (\frac{1}{\sqrt{N_r}}, \frac{1}{\sqrt{N_r}}, \ldots)$ with step size $\alpha = 1$.

3 Hybrid precoding and analog combining design

How to design the hybrid precoding and analog combining jointly?

Joint Hybrid Precoding and Analog Combining Scheme

The design of analog precoding matrix

As shown that one ϕ_i cannot perfectly match different values of $\theta_{i,l}, 1 \leq l \leq N_t$. Therefore, the residue of phase elimination may still exist.
To this end, we apply the analog precoding by construct the intermediate channel as:

$$\mathbf{H}_{int} = \begin{bmatrix} \mathbf{w}_1^H \mathbf{H}_1 \\ \vdots \\ \mathbf{w}_K^H \mathbf{H}_K \end{bmatrix} \in \mathbb{C}^{K \times N_t}$$

Based on it, we construct the analog precoding matrix by the phase elimination, that is:

$$\mathbf{F}(l,k) = \frac{1}{\sqrt{N_t}} e^{j\psi_{l,k}}, \forall l, k,$$

$\psi_{l,k}$ is the phase of the (l, k)-th element of the conjugate transpose of $\boldsymbol{H}_{int}$.

Obviously, the analog precoding matrix is designed jointly with the analog combining vectors to eliminate the phases of the channel matrix.

The design of digital precoding matrix

We define the equivalent channel vetor of the k-th user as:

$$\tilde{\mathbf{h}}_k = \mathbf{w}_k^H \mathbf{H}_k \mathbf{F}.$$

We construct the complementary channel matrix of the k-th user:

$$\overline{\mathbf{H}}_k = \left[\tilde{\mathbf{h}}_1^T, \ldots, \tilde{\mathbf{h}}_{k-1}^T, \tilde{\mathbf{h}}_{k+1}^T, \ldots, \tilde{\mathbf{h}}_K^T\right]^T.$$

To eliminate the inter-user interference, the k-th column of $\mathbf{D}$ should lie in the null space of $\overline{\mathbf{H}}_k$.

Therefore, we perform the singular value decomposition :

$$\overline{\mathbf{H}}_k = \overline{\mathbf{U}}_k \overline{\mathbf{\Sigma}}_k \left[\overline{\mathbf{V}}_k^{K-1}, \overline{\mathbf{v}}_k\right]^H,$$

The last right singular vector $\overline{\mathbf{v}}_k$ is the null space of $\overline{\mathbf{H}}_k$, so that we can construct the k-th column of $\mathbf{D}$ as:

$$\mathbf{d}^k = \overline{\mathbf{v}}_k.$$

4 Complexity Analysis

The computation of the analog combiner

- compute $\mathbf{q}_k^*$: $\mathcal{O}(N_tN_r)$
- compute the gradient: $\mathcal{O}(2N_tN_r)$
- the overall complexity with the number of iteration r: $\mathcal{O}\big((2r+1)N_tN_r\big)$

The computation of the analog precoder

- compute the intermediate channel: $\mathcal{O}(KN_tN_r)$

The computation of the digital precoder

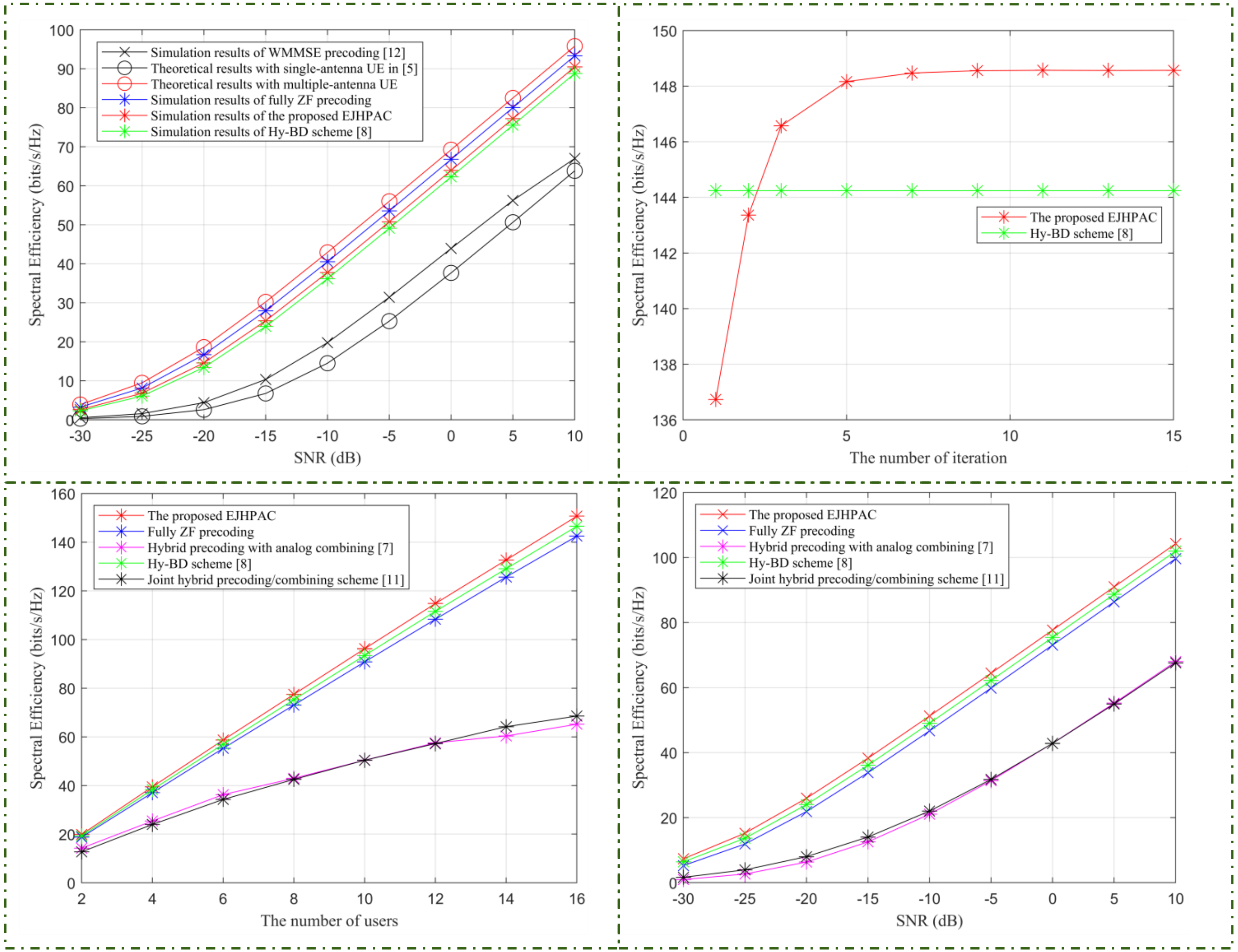
- compute the equivalent channel and the SVD operation: $\mathcal{O}\big(K[(K+1)N_tN_r+K^2(K-1)]\big)$

THE COMPUTATIONAL COMPLEXITIES OF EJHPAC AND OTHER ALGORITHMS

	Overall complexity	64×16 4-user massive MIMO	256×16 8-user massive MIMO
EJHPAC	$\mathcal{O}(K[(2r+K+3)N_tN_r+K^2(K-1)])$	$53440(r=3)$	$691712(r=5)$
Hy-BD in [8]	$\mathcal{O}(K[(N_r+K+2)N_tN_r+K^2(K-1)])$	90304	855552
Joint scheme in [11]	$\mathcal{O}(K(N_t^2N_r+4N_t^2+N_r^2))$	328704	10487808

The proposed algorithm is efficient on computational complexity.

5 Simulations



- Multiple receive antennas introduce the improvement on performance.
- The simulation results of EJHPAC is close to the derived theoretical spectral efficiency.
- EJHPAC can effectively eliminate the phases in mmWave channels.
- EJHPAC shows higher spectral efficiency because of obtaining larger power gain in the analog domain.

Conclusion

Propose an efficient joint hybrid precoding and analog combining scheme named as EJHPAC for massive MIMO systems with multiple-antenna UE.

According to theoretical analysis of the spectral efficiency, the problem of analog combining is transformed and the GDP method is chosen to solve it.	**The analog precoding is jointly designed by phase elimination to achieve the power gain while the digital precoding is designed to eliminate interference.**	**The complexity analysis indicates that EJHPAC is efficient as well. Finally, simulation results confirm that EJHPAC attains better performance than other algorithms.**

Thank you for your watching

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